

**3-4 Study Guide and Intervention**

**Systems of Equations in Three Variables**

**Systems in Three Variables** Use the methods used for solving systems of linear equations in two variables to solve systems of equations in three variables. A system of three equations in three variables can have a unique solution, infinitely many solutions, or no solution. A solution is an ordered triple.

**Example**

Solve the system of equations. 

$$3x + y - z = -6$$

$$2x - y + 2z = 8$$

$$4x + y - 3z = -21$$

**Step 1** Use elimination to make a system of two equations in two variables.

$$3x + y - z = -6 \quad \text{First equation}$$

$$2x - y + 2z = 8 \quad \text{Second equation}$$

$$4x + y - 3z = -21 \quad \text{Third equation}$$

Add to eliminate $y$.

$$5x + z = 2$$

**Step 2** Solve the system of two equations.

$$5x + z = 2$$

$$6x - z = -13$$

Add to eliminate $z$.

$$11x = -11$$

Divide both sides by 11.

$$x = -1$$

Substitute $-1$ for $x$ in one of the equations with two variables and solve for $z$.

$$5x + z = 2$$

$$5(-1) + z = 2$$

Replace $x$ with $-1$.

$$-5 + z = 2$$

Multiply.

$$z = 7$$

Add 5 to both sides.

The result so far is $x = -1$ and $z = 7$.

**Step 3** Substitute $-1$ for $x$ and $7$ for $z$ in one of the original equations with three variables.

$$3x + y - z = -6$$

Original equation with three variables

$$3(-1) + y - 7 = -6$$

Replace $x$ with $-1$ and $z$ with $7$.

$$-3 + y - 7 = -6$$

Multiply.

$$y = 4$$

Simplify.

The solution is $(-1, 4, 7)$.

**Exercises**

Solve each system of equations.

1. $2x + 3y - z = 0$

   $x - 2y - 4z = 14$

   $3x + y - 8z = 17$

   $(4, -3, -1)$

2. $2x - y + 4z = 11$

   $x + 2y - 6z = -11$

   $3x - 2y - 10z = 11$

   $(2, -5, \frac{1}{2})$

3. $x - 2y + z = 8$

   $2x + y - z = 0$

   $3x - 6y + 3z = 24$

   infinitely many solutions

4. $3x - y - z = 5$

   $3x + 2y - z = 11$

   $6x - 3y + 2z = -12$

   $\left(\frac{2}{3}, 2, -5\right)$

   no solution

5. $2x - 4y - z = 10$

   $4x - 8y - 2z = 16$

   $3x + y + z = 12$

   infinitely many solutions

6. $x - 6y + 4z = 2$

   $2x + 4y - 8z = 16$

   $x - 2y = 5$

   infinitely many solutions
3-4 Study Guide and Intervention (continued)

Systems of Equations in Three Variables

Real-World Problems

Example The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases for $99 on Monday. On Tuesday they sold 4 balls, 8 bats, and 2 bases for $78. On Wednesday they sold 2 balls, 3 bats, and 1 base for $33.60. What are the prices of 1 ball, 1 bat, and 1 base?

First define the variables.

\( x = \) price of 1 ball
\( y = \) price of 1 bat
\( z = \) price of 1 base

Translate the information in the problem into three equations.

\[10x + 3y + 2z = 99\]
\[4x + 8y + 2z = 78\]
\[2x + 3y + z = 33.60\]

Subtract the second equation from the first equation to eliminate \( z \).

\[
\begin{align*}
10x + 3y + 2z &= 99 \\
(-) 4x + 8y + 2z &= 78 \\
6x - 5y &= 21
\end{align*}
\]

Multiply the third equation by 2 and subtract from the second equation.

\[
\begin{align*}
4x + 8y + 2z &= 78 \\
(-) 4x + 6y + 2z &= 67.20 \\
2y &= 10.80 \\
y &= 5.40
\end{align*}
\]

Substitute 5.40 for \( y \) in the equation

\[6x - 5y = 21\]

\[6x - 5(5.40) = 21\]

\[6x = 48\]

\[x = 8\]

Substitute 8 for \( x \) and 5.40 for \( y \) in one of the original equations to solve for \( z \).

\[
\begin{align*}
10x + 3y + 2z &= 99 \\
10(8) + 3(5.40) + 2z &= 99 \\
80 + 16.20 + 2z &= 99 \\
2z &= 2.80 \\
z &= 1.40
\end{align*}
\]

So a ball costs $8, a bat $5.40, and a base $1.40.

Exercises

1. FITNESS TRAINING Carly is training for a triathlon. In her training routine each week, she runs 7 times as far as she swims, and she bikes 3 times as far as she runs. One week she trained a total of 232 miles. How far did she run that week? **56 miles**

2. ENTERTAINMENT At the arcade, Ryan, Sara, and Tim played video racing games, pinball, and air hockey. Ryan spent $6 for 6 racing games, 2 pinball games, and 1 game of air hockey. Sara spent $12 for 3 racing games, 4 pinball games, and 5 games of air hockey. Tim spent $12.25 for 2 racing games, 7 pinball games, and 4 games of air hockey. How much did each of the games cost? **racing game: $0.50; pinball: $0.75; air hockey: $1.50**

3. FOOD A natural food store makes its own brand of trail mix out of dried apples, raisins, and peanuts. One pound of the mixture costs $3.18. It contains twice as much peanuts by weight as apples. One pound of dried apples costs $4.48, a pound of raisins $2.40, and a pound of peanuts $3.44. How many ounces of each ingredient are contained in 1 pound of the trail mix? **3 oz of apples, 7 oz of raisins, 6 oz of peanuts**