**Solving Quadratic Equations by Factoring**

**Factored Form** To write a quadratic equation with roots \( p \) and \( q \), let \((x - p)(x - q) = 0\). Then multiply using FOIL.

**Example** Write a quadratic equation in standard form with the given roots.

a. \(3, -5\)

\[
(x - p)(x - q) = 0 \quad \text{Write the pattern.}
\]

\[
(x - 3)(x + 5) = 0 \quad \text{Replace } p \text{ with } 3, q \text{ with } -5.
\]

\[
x^2 + 2x - 15 = 0 \quad \text{Use FOIL.}
\]

The equation \(x^2 + 2x - 15 = 0\) has roots 3 and -5.

b. \(-\frac{7}{8}, \frac{1}{3}\)

\[
(x - p)(x - q) = 0
\]

\[
\left[ x - \left(-\frac{7}{8}\right) \right] \left( x - \frac{1}{3} \right) = 0
\]

\[
\left( x + \frac{7}{8} \right) \left( x - \frac{1}{3} \right) = 0
\]

\[
\frac{24}{8} \cdot \left( 8x + 7 \left( 3x - 1 \right) \right) = 24 \cdot 0
\]

\[
24x^2 + 13x - 7 = 0
\]

The equation \(24x^2 + 13x - 7 = 0\) has roots \(-\frac{7}{8}\) and \(\frac{1}{3}\).

**Exercises**

Write a quadratic equation in standard form with the given root(s).

1. \(3, -4\)

\[x^2 + x - 12 = 0\]

2. \(-8, -2\)

\[x^2 + 10x + 16 = 0\]

3. \(1, 9\)

\[x^2 - 10x + 9 = 0\]

4. \(-5\)

\[x^2 + 10x + 25 = 0\]

5. \(10, 7\)

\[x^2 - 17x + 70 = 0\]

6. \(-2, 15\)

\[x^2 - 13x - 30 = 0\]

7. \(-\frac{1}{3}, 5\)

\[3x^2 - 14x - 5 = 0\]

8. \(2, \frac{2}{3}\)

\[3x^2 - 8x + 4 = 0\]

9. \(-7, \frac{3}{4}\)

\[4x^2 + 25x - 21 = 0\]

10. \(3, \frac{2}{5}\)

\[5x^2 - 17x + 6 = 0\]

11. \(-\frac{4}{9}, -1\)

\[9x^2 + 13x + 4 = 0\]

12. \(9, \frac{1}{6}\)

\[6x^2 - 55x + 9 = 0\]

13. \(\frac{2}{3}, \frac{2}{3}\)

\[9x^2 - 4 = 0\]

14. \(\frac{5}{4}, \frac{1}{2}\)

\[8x^2 - 6x - 5 = 0\]

15. \(\frac{3}{7}, \frac{1}{5}\)

\[35x^2 - 22x + 3 = 0\]

16. \(-\frac{7}{8}, \frac{7}{2}\)

\[16x^2 - 42x - 49 = 0\]

17. \(\frac{1}{2}, \frac{3}{4}\)

\[8x^2 - 10x + 3 = 0\]

18. \(\frac{1}{8}, \frac{1}{6}\)

\[48x^2 - 14x + 1 = 0\]
Chapter 4

Solve Equations by Factoring

When you use factoring to solve a quadratic equation, you use the following property.

**Zero Product Property**
For any real numbers \(a\) and \(b\), if \(ab = 0\), then either \(a = 0\) or \(b = 0\), or both \(a\) and \(b = 0\).

**Example**

Solve each equation by factoring.

**a.** \(3x^2 = 15x\)

- \(3x^2 = 15x\)  Original equation
- \(3x^2 - 15x = 0\)  Subtract 15x from both sides.
- \(3x(x - 5) = 0\)  Factor the binomial.
- \(3x = 0\) or \(x - 5 = 0\)  Zero Product Property
- \(x = 0\) or \(x = 5\)  Solve each equation.

The solution set is \(\{0, 5\}\).

**b.** \(4x^2 - 5x = 21\)

- \(4x^2 - 5x - 21 = 0\)  Original equation
- \((4x + 7)(x - 3) = 0\)  Factor the trinomial.
- \(4x + 7 = 0\) or \(x - 3 = 0\)  Zero Product Property
- \(x = -\frac{7}{4}\) or \(x = 3\)  Solve each equation.

The solution set is \(\left\{-\frac{7}{4}, 3\right\}\).

**Exercises**

Solve each equation by factoring.

1. \(6x^2 - 2x = 0\)
   - \(\left\{\frac{1}{3}, 0\right\}\)
2. \(x^2 = 7x\)
   - \(\{0, 7\}\)
3. \(20x^2 = -25x\)
   - \(\left\{0, -\frac{5}{4}\right\}\)
4. \(6x^2 = 7x\)
   - \(\left\{\frac{7}{6}, 0\right\}\)
5. \(6x^2 - 27x = 0\)
   - \(\left\{\frac{9}{2}, 0\right\}\)
6. \(12x^2 - 8x = 0\)
   - \(\left\{\frac{2}{3}, 0\right\}\)
7. \(x^2 + x - 30 = 0\)
   - \(\{5, -6\}\)
8. \(2x^2 - x - 3 = 0\)
   - \(\left\{\frac{3}{2}, -1\right\}\)
9. \(x^2 + 14x + 33 = 0\)
   - \(\{-11, -3\}\)
10. \(4x^2 + 27x - 7 = 0\)
    - \(\left\{\frac{1}{4}, -7\right\}\)
11. \(3x^2 + 29x - 10 = 0\)
    - \(\left\{-\frac{1}{3}, -10\right\}\)
12. \(6x^2 - 5x - 4 = 0\)
    - \(\left\{-\frac{4}{3}, 1\right\}\)
13. \(12x^2 - 8x + 1 = 0\)
    - \(\left\{\frac{1}{6}, \frac{1}{2}\right\}\)
14. \(5x^2 + 28x - 12 = 0\)
    - \(\left\{\frac{2}{5}, -6\right\}\)
15. \(2x^2 - 250x + 5000 = 0\)
    - \(\{100, 25\}\)
16. \(2x^2 - 11x - 40 = 0\)
    - \(\left\{8, -\frac{5}{2}\right\}\)
17. \(2x^2 + 21x - 11 = 0\)
    - \(\left\{-11, \frac{1}{2}\right\}\)
18. \(3x^2 + 2x - 21 = 0\)
    - \(\left\{\frac{7}{3}, -3\right\}\)
19. \(8x^2 - 14x + 3 = 0\)
    - \(\left\{\frac{1}{2}, \frac{3}{4}\right\}\)
20. \(6x^2 + 11x - 2 = 0\)
    - \(\left\{-2, \frac{1}{6}\right\}\)
21. \(5x^2 + 17x - 12 = 0\)
    - \(\left\{\frac{3}{5}, -4\right\}\)
22. \(12x^2 + 25x + 12 = 0\)
    - \(\left\{-\frac{4}{3}, -\frac{3}{4}\right\}\)
23. \(12x^2 + 18x + 6 = 0\)
    - \(\left\{\frac{1}{2}, -1\right\}\)
24. \(7x^2 - 36x + 5 = 0\)
    - \(\left\{1, \frac{5}{7}\right\}\)